Chiral symmetry on a lattice with hopping interactions

Takanori Sugihara

RIKEN BNL Research Center

Brookhaven National Laboratory

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Introduction

Problems of lattice fermion

- Species doubling
- Deviation of the propagator

Staggered, SLAC, Wilson, Kaplan, and Overlap.

Introduction

Problems of lattice fermion

- Species doubling
- Deviation of the propagator

This talk

- Lanczos factor
- Ultralocal hopping interactions

Hybrid of Wilson and SLAC approaches

Momentum-space rep. on a lattice

Continuum theory for (1+1)-d Dirac spinor

$$H = -i \int dx \bar{\psi} \gamma^1 \partial_1 \psi$$

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Momentum-space rep. on a lattice

$$H = \sum_{l=-N/2+1}^{N/2} p_l \bar{\zeta}_l \gamma^1 \zeta_l, \quad p_l = \frac{2\pi l}{N}$$

Discrete Fourier trans. $\psi_n = \frac{1}{\sqrt{N}} \sum_l e^{i2\pi l n/N} \zeta_l$

What is the real-space rep of p?

Consider a function

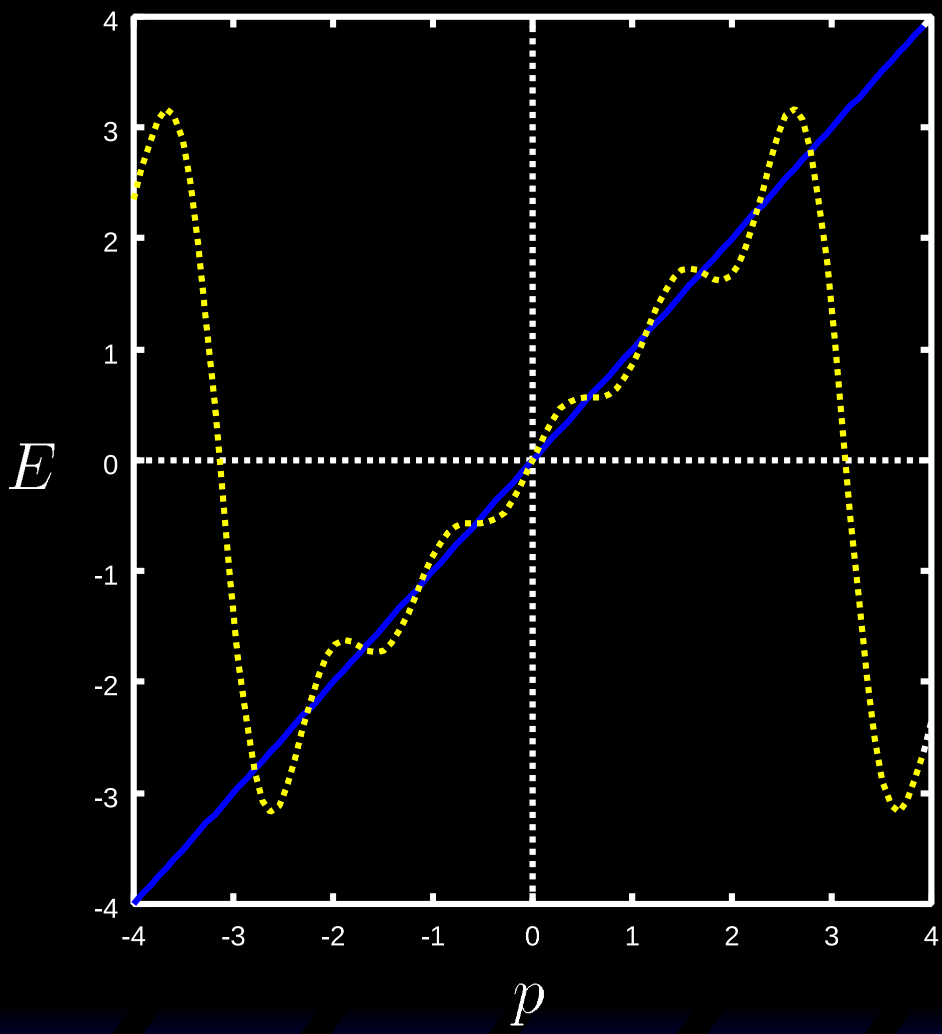
$$s(p) \equiv \sum_{\alpha=1}^{M} \frac{2(-1)^{\alpha-1}}{\alpha} \sin(\alpha p)$$

In the limit $M \to \infty$

$$p = \lim_{M\to\infty} s(p)$$

Periodicity of $2\pi \to$ Doubler at $|p| = \pi$.

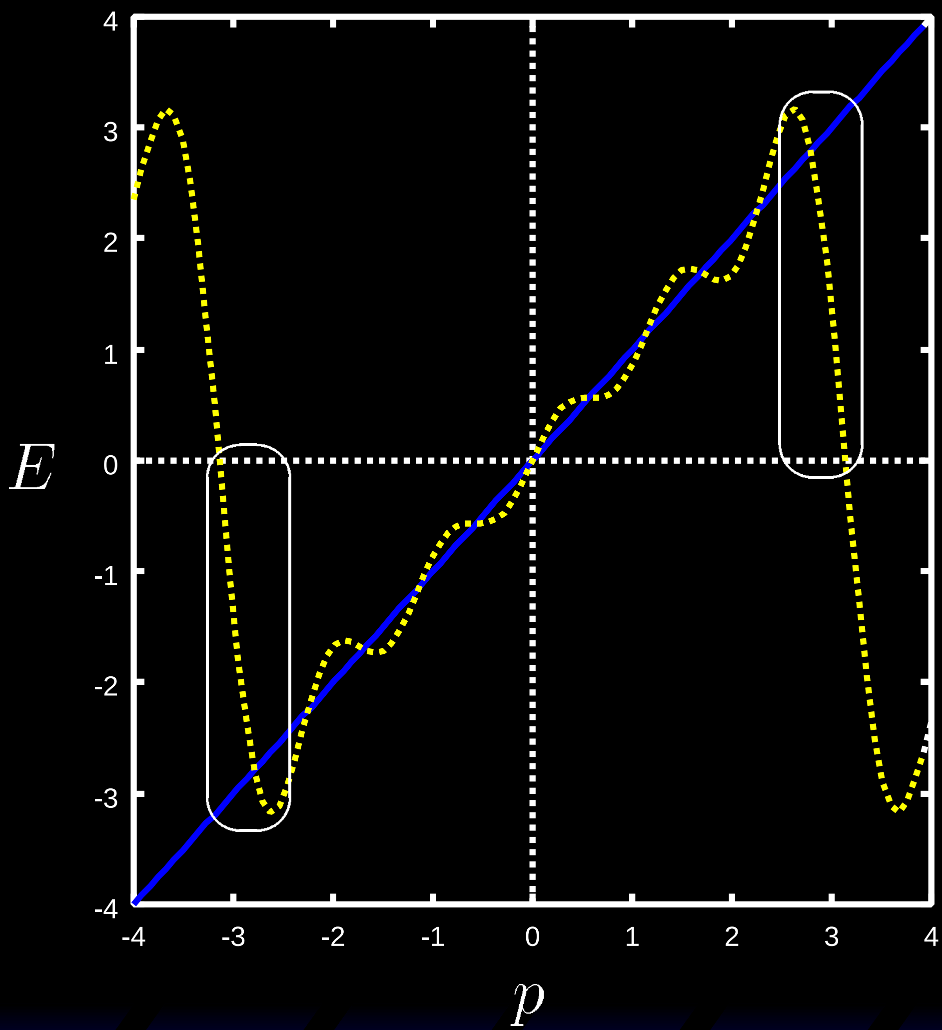
Energy of one-particle state



Blue: continuum p

Yellow: $s(p)$ with $M = 5$

Energy of one-particle state



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In addition to usual doubler at the momentum boundary

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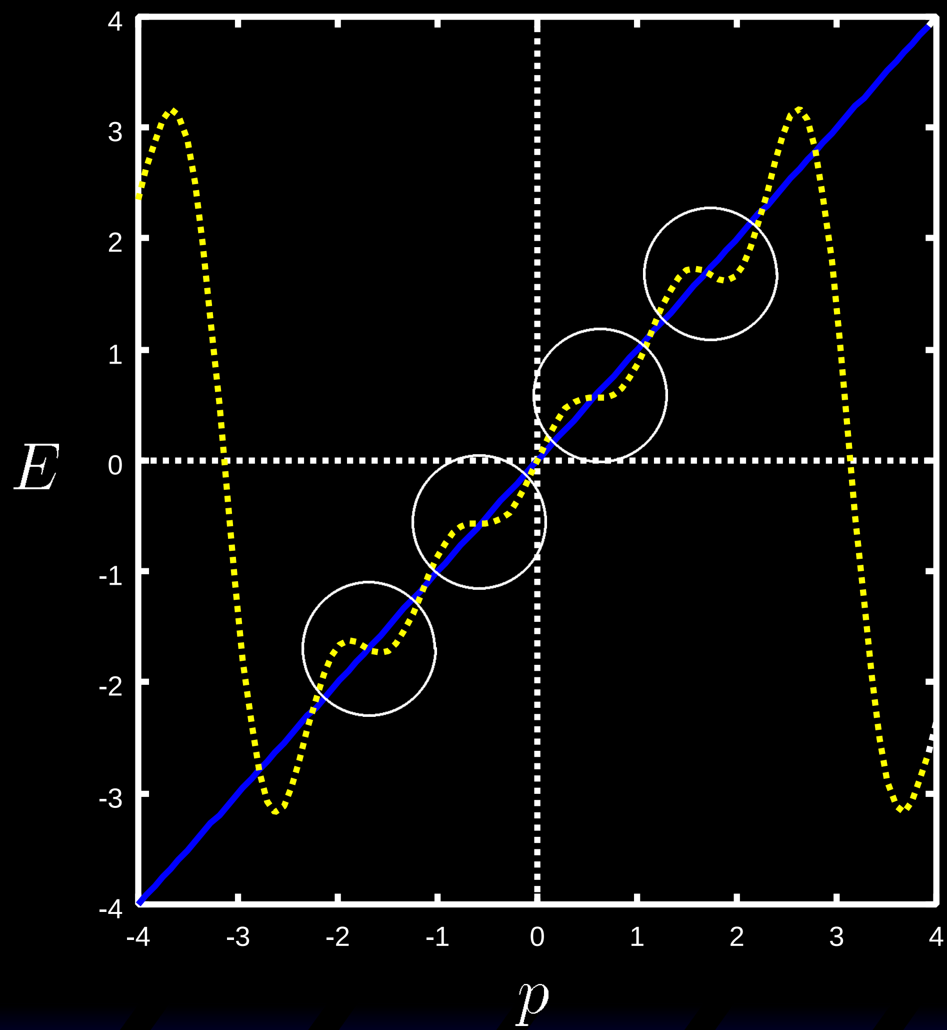

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Truncation with a finite M causes oscillation around the correct result.

Two types of doublers

- Jump at the boundary $|p| = \pi$
- Oscillation (Gibbs phenomenon)

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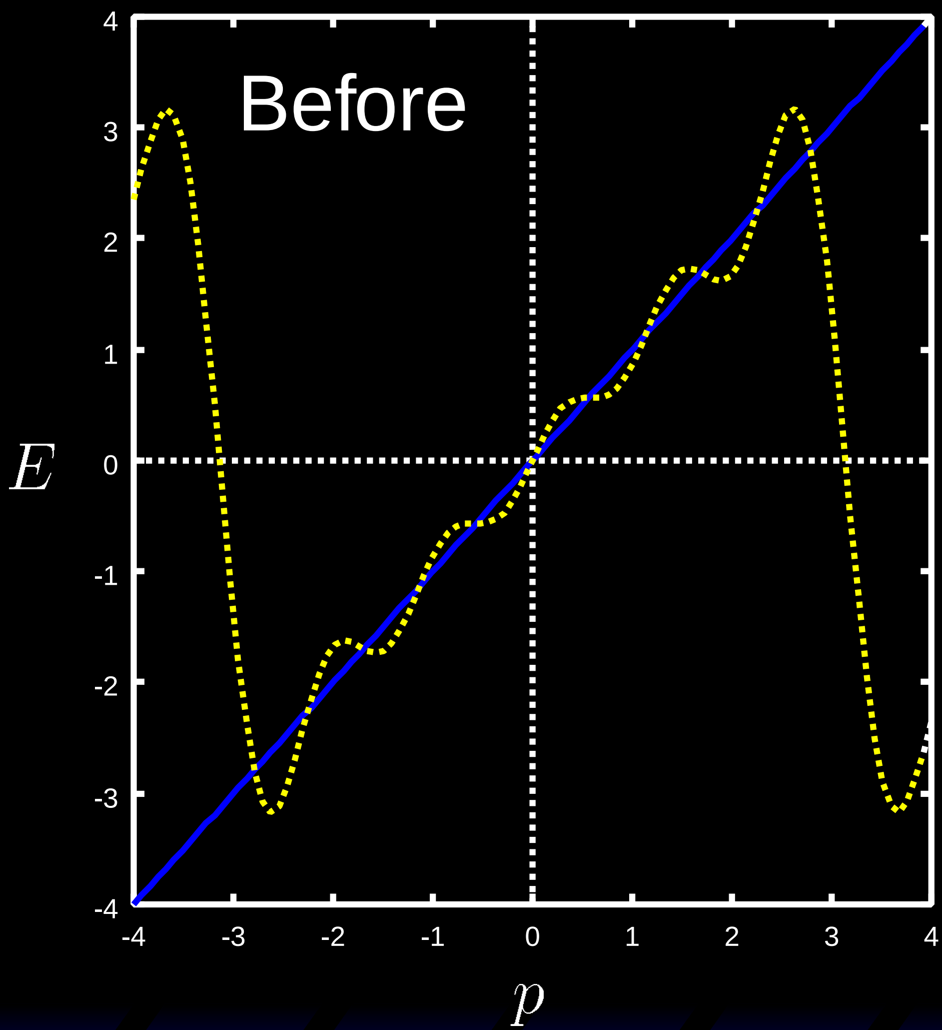
Modify the coefficients

$$s(p) = \sum_{\alpha=1}^{M} F_\alpha \frac{2(-1)^{\alpha-1}}{\alpha} \sin(\alpha p)$$

Lanczos factor: $F_\alpha = \frac{M+1}{\pi\alpha} \sin\left(\frac{\pi\alpha}{M+1}\right)$

Energy of one-particle state



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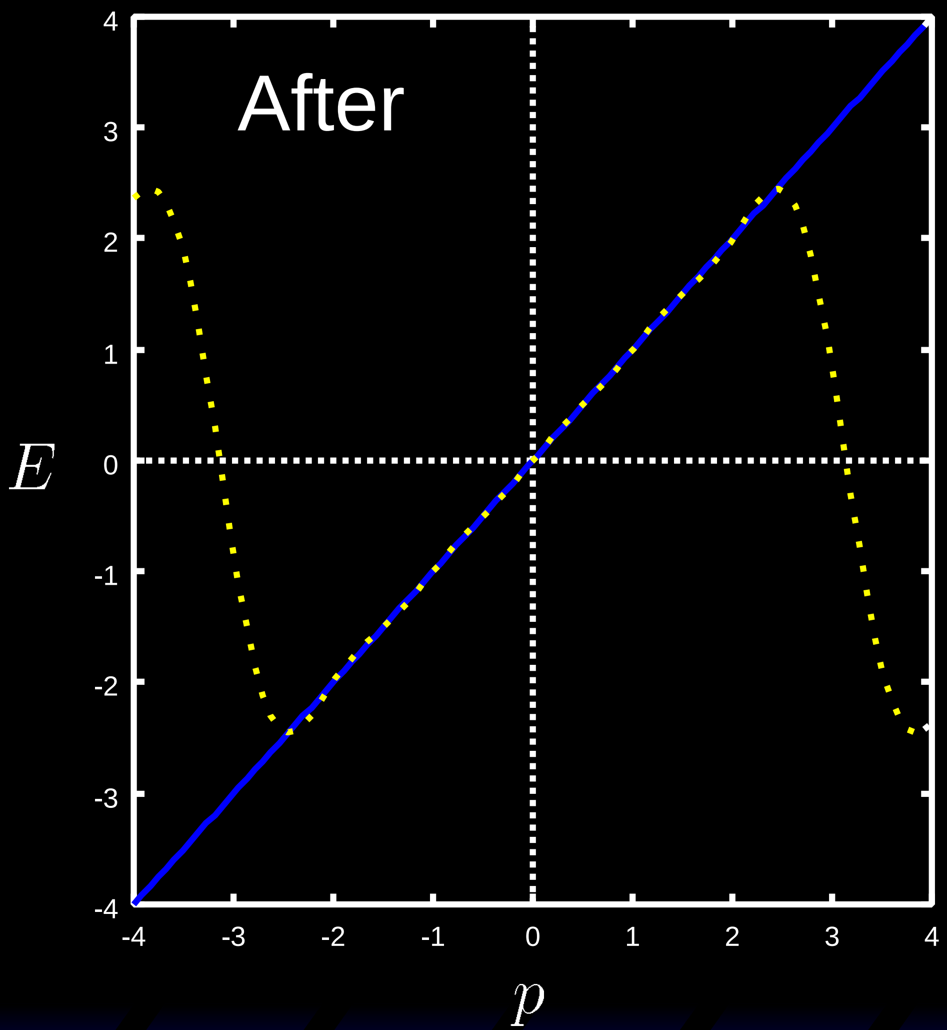
The Lanczos factor removes the oscillation.

Energy of one-particle state



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The Lanczos factor removes the oscillation.

Removal of the doubler at $|p| = \pi$

Momentum-space Hamiltonian ($s_l \equiv s(p_l)$)

$$H = \sum_l \zeta_l^\dagger \begin{pmatrix} s_l & 0 \\ 0 & -s_l \end{pmatrix} \zeta_l$$

Removal of the doubler at $|p| = \pi$

Momentum-space Hamiltonian ($s_l \equiv s(p_l)$)

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Introduce interactions to remove the doubler

$$H = \sum_l \zeta_l^\dagger \begin{pmatrix} s_l & c_l \\ c_l & -s_l \end{pmatrix} \zeta_l$$

Wilson-like interactions c_l

Diagonalization of H

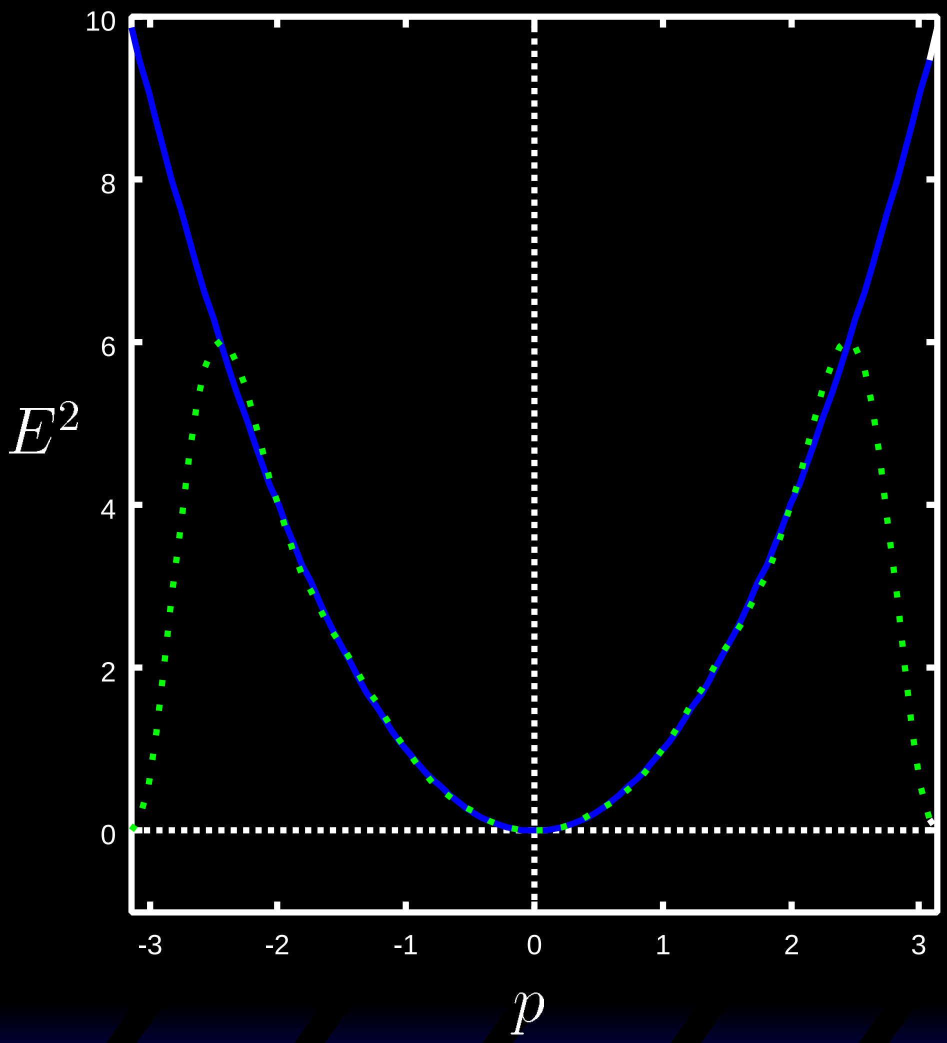
$$H = \sum_l \zeta_l'^\dagger \begin{pmatrix} \sqrt{s_l^2 + c_l^2} & 0 \\ 0 & -\sqrt{s_l^2 + c_l^2} \end{pmatrix} \zeta_l'$$

Use c_l to remove the doubler.

$$c(p) = \frac{C_0}{2} + \sum_{\alpha=1}^{M} C_\alpha \cos(\alpha p)$$

where $c_l \equiv c(p_l)$.

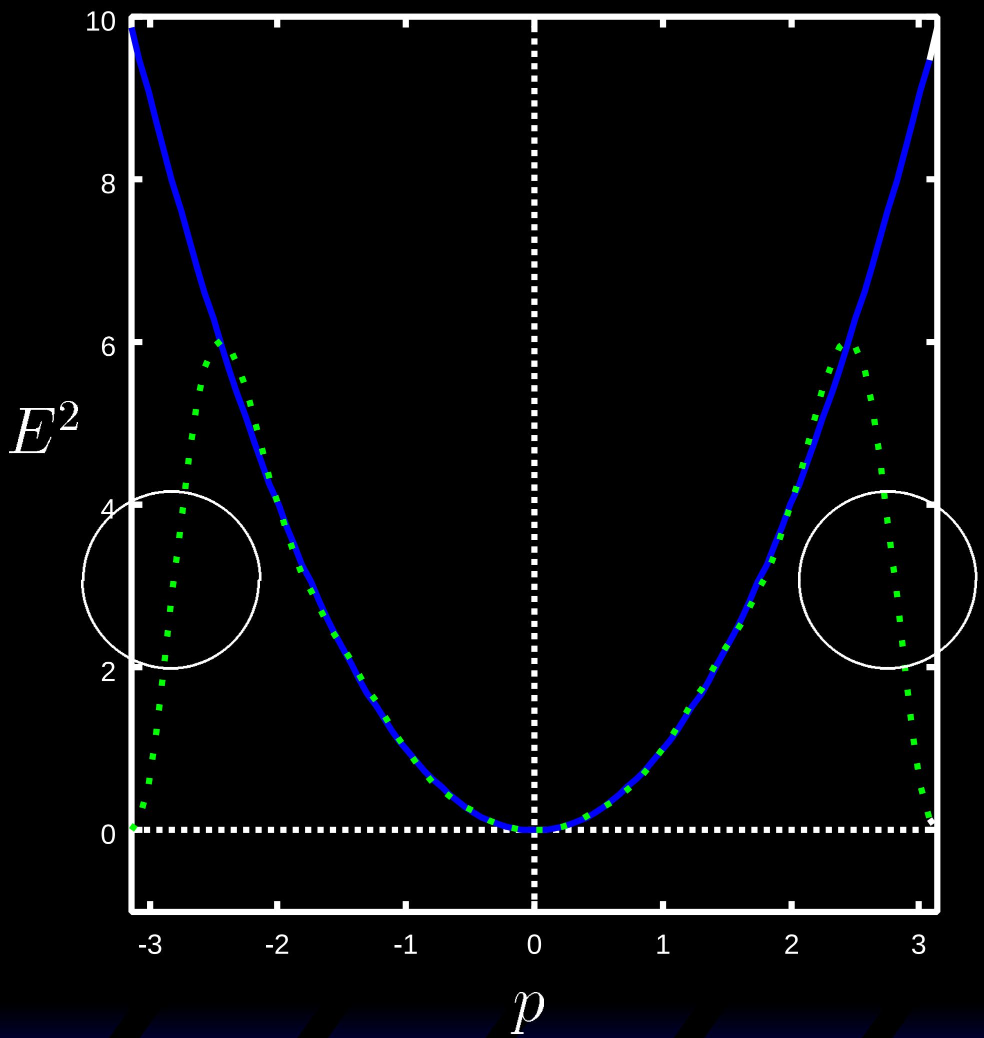
Energy squared vs momentum



Blue: continuum p^2

Green: $s^2(p)$

Energy squared vs momentum



Blue: continuum p^2

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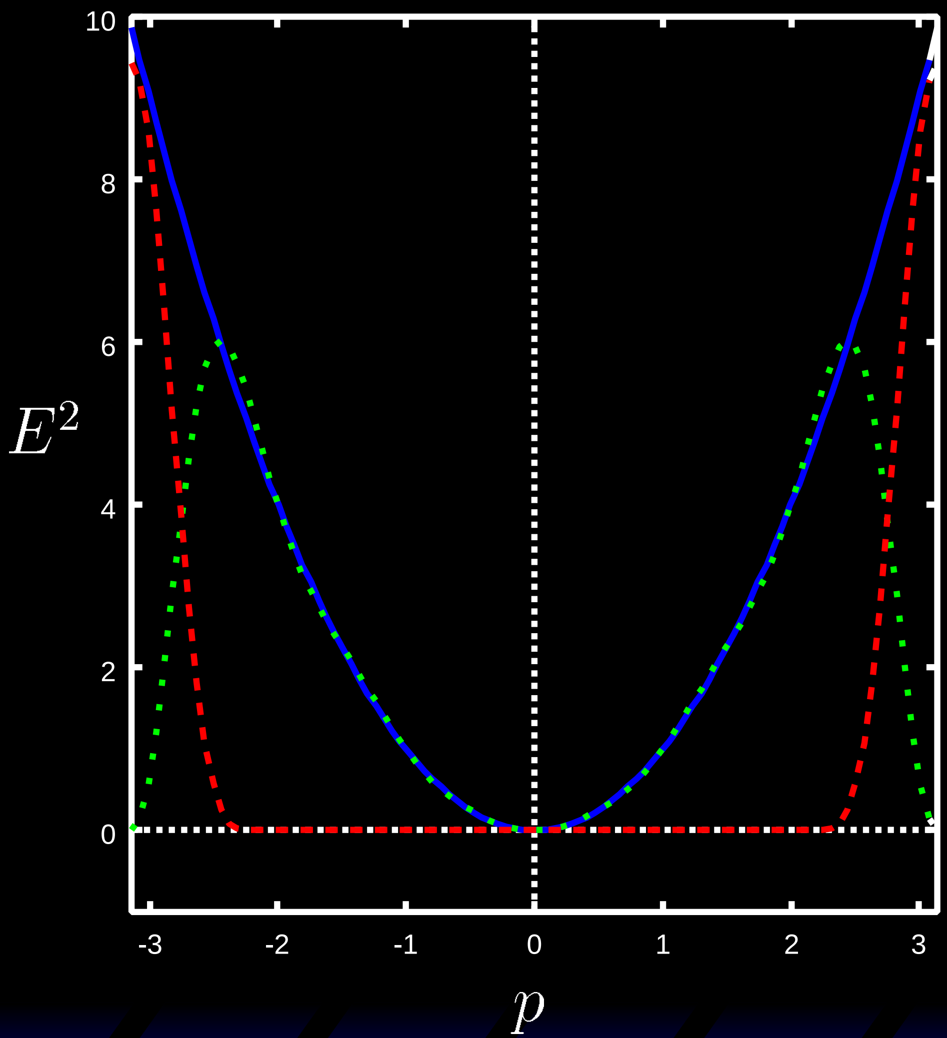
Raise the hemlines

Energy squared vs momentum

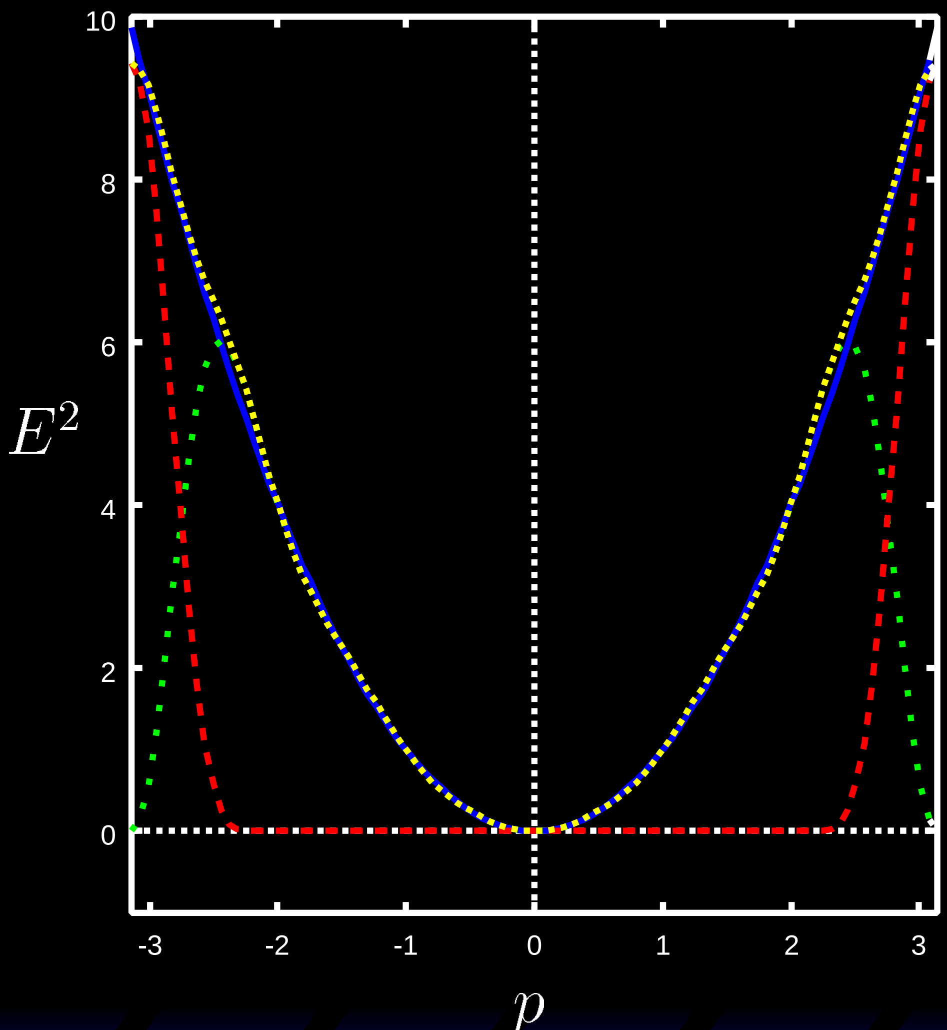


Blue: continuum p^2

Green: $s^2(p)$

Red: $c^2(p)$

Energy squared vs momentum



Blue: continuum p^2

Green: $s^2(p)$

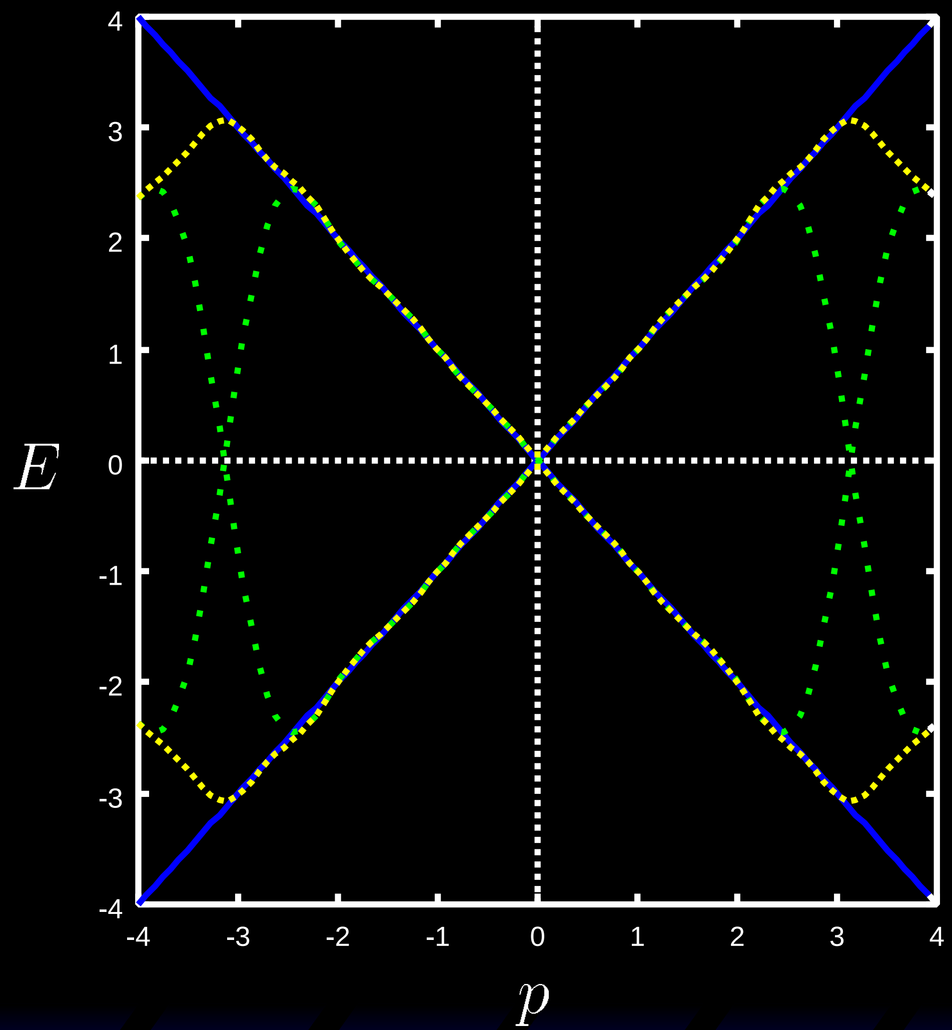
Red: $c^2(p)$

Yellow: Green+Red

Yellow is almost p^2.

The doubler removed.

Energy vs momentum



Blue: continuum $\pm p$

Green: $\pm s(p)$

Yellow: $\pm\sqrt{s^2(p) + c^2(p)}$

Good agreement except for a small deviation at $|p| = 2.3$.

Chiral charge Q_5

In the new basis, γ_5 becomes

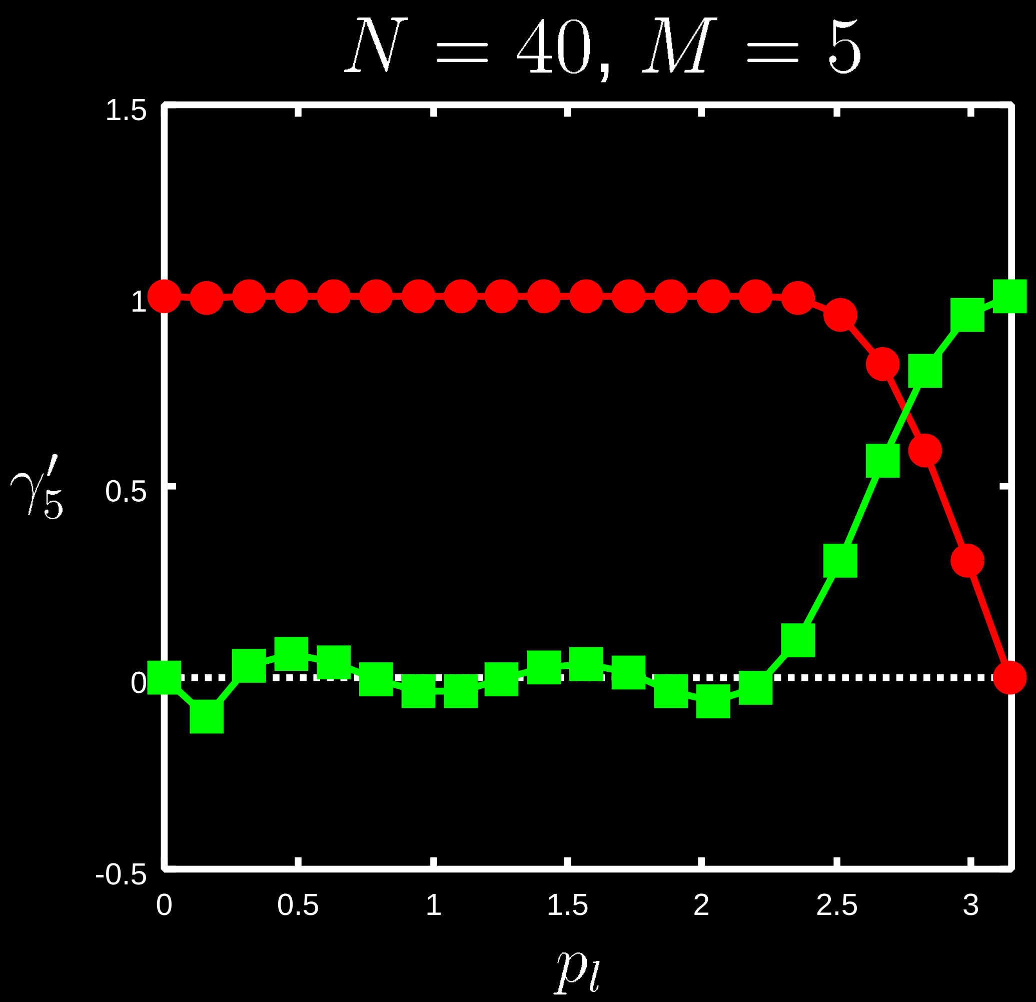
$$\gamma_5' = \frac{s_l + k_l}{k_l^2 + s_l k_l} \begin{pmatrix} s_l & -c_l \\ -c_l & -s_l \end{pmatrix}$$

for $l > 0$

$$\gamma_5' = \frac{-s_l + k_l}{k_l^2 - s_l k_l} \begin{pmatrix} s_l & c_l \\ c_l & -s_l \end{pmatrix}$$

for $l < 0$, and $\gamma_5' = \gamma_5$ for $l = 0$.

Chiral property



Red: $(\gamma_5')_{1,1}$

Green: $-(\gamma_5')_{1,2}$

$$[H, Q_5] \sim 0$$

Approximate chiral sym. at low energy.

Real-space Hamiltonian

Real-space representation for gauge theory

$$H = \sum_{n=1}^{N} \left\{ \frac{1}{2a} \sum_{\alpha=1}^{M} \left[iS_\alpha (\bar{\psi}_{n+\alpha} \gamma^1 \psi_n - \bar{\psi}_n \gamma^1 \psi_{n+\alpha}) \right.\right.$$

$$\left.\left. + C_\alpha (\bar{\psi}_{n+\alpha} \psi_n + \bar{\psi}_n \psi_{n+\alpha}) \right] + \left(m + \frac{C_0}{2a} \right) \bar{\psi}_n \psi_n \right\}$$

Take care of doubler of each direction
to extend the method to higher dimensions.

Euclidean action

For Monte-Carlo analysis

$$S_{\mathrm{E}} = \sum_n \left\{ \frac{1}{2a} \sum_{\alpha=1}^{M} \sum_{\mu=1}^{2} \left[S_\alpha \left(\bar{\psi}_n \gamma_\mu \psi_{n+\alpha\hat{\mu}} - \bar{\psi}_{n+\alpha\hat{\mu}} \gamma_\mu \psi_n \right) \right. \right.$$

$$\left. \left. + C_\alpha \left(\bar{\psi}_n \psi_{n+\alpha\hat{\mu}} + \bar{\psi}_{n+\alpha\hat{\mu}} \psi_n \right) \right] + \left(m + \frac{C_0}{a} \right) \bar{\psi}_n \psi_n \right\}$$

The continuum limit $a \to 0$ is taken with the parameter M fixed.

Conclusion

Explicit breaking of chiral symmetry has been compressed to high energy with the Lanczos factor and ultralocal hopping interactions.

- Chiral symmetry with small M
- Good agreement with the continuum
- Systematic improvement with M

Check if gauge fields affect chiral properties.